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MATHEMATICAL EXPLANATION AND COMPLEX SYSTEMS

Complex systems present direct challenges to a number of positions in the philosophy of science on prediction, representation and explanation. Some examples of complex systems include stock markets, the weather and physical phenomena like superconductivity. The thematic goal of this paper involves examining the relation between the mathematics used to treat complex systems, understood as a formal representational tool, and its role in providing a 'physical' understanding of these systems. More specifically, I intend to address the way that a mathematical technique known as renormalization group (RG) methods can provide an explanatory foundation which highlights structural features that different complex systems have in common. What this suggests is that there is a fundamental level of explanation underlying complex systems which can be explicated via the mathematics of RG.

Introduction and Background

To my mind one of the most difficult challenges in the philosophy of science involves analysing the role of mathematical explanation in the behaviour of complex systems or systems that display emergent behaviour or phenomena. The problem has many facets but the one that is especially interesting is how a particular mathematical method, known as the renormalization group (RG), can be used not just as a way of accounting for behaviour in different types of physical systems or calculating values for particular parameters, functions commonly ascribed to mathematics, but to furnish genuine *physical* understanding and explanation. Physical phenomena as diverse as those found in quantum field theory (QFT), financial markets, statistical physics, as well as the dynamics of biological evolution, all use the mathematics of RG in explaining and predicting how these various systems behave under certain constraints. Despite this common methodology there has been very little systematic analysis of the possible underlying features that might explain the success of RG in such disparate areas, features that stem from the general aspects of the mathematics itself, possible similarities among the phenomena, or a combination of both.

Addressing this problem involves two interrelated areas in the philosophy of science and the philosophy of mathematics: (1) the nature and status of mathematical explanation with respect to RG, and (2) how this type of explanation can illustrate general *structural* features of complex systems that make them amenable to the same mathematical treatment. My hypothesis is that a clearer delineation of the criteria that characterise mathematical explanations associated with RG methods (e.g. structural similarity, among others) in the treatment of complex systems, specifically statistical mechanics and financial markets (an area now known as econophysics), will enable us to appreciate the different ways that mathematics can provide physical, qualitative understanding.

What makes complex systems interesting from a philosophical perspective is that they seem to defy the kinds of explanatory and predictive frameworks (specifically reduction) that are applicable in other physical contexts. This is due in part to the fact that while the behaviour of complex systems involves many interacting constituents, that behaviour cannot be explained in terms of those interactions. Examples include IT networks, ecosystems, brains, stock markets, weather and physical phenomena like superconductivity, systems that present some of the most pressing real-world challenges for society, government and industry – in the environment, health and medicine, finance and economics, population growth, technology and transport. Little attention has been paid in the philosophical literature to structural features that various complex systems might have in common, features that can give rise to explanatory strategies that might enable us to understand better general features of their behaviour. Nor has much attention been paid to the ways that RG methods are used in treating various kinds of complex systems and how these methods might provide a general explanatory framework that incorporates some notion of structural similarity. As Goldenfeld and Kadanoff (1999), two leading theorists in the foundations of RG methods, remark, complexity means we have structure with variations. Hence, the search for general explanatory principles that illuminate these structural features is extremely important.

A way of distinguishing complex systems from very complicated ones is that they exhibit self-organisation and emergence. Emergence is the appearance of behaviour that could not be predicted, reduced to or explained from knowledge of the system's constituents; self-organisation typically means there is no external force required for producing these emergent features (Gitterman and Halpern, 2004; Batterman, 2002). A consequence of emergence and self-organisation is that certain global features of complex systems are largely independent of particular realisations of their parts. The important philosophical questions concern the type of explanatory framework we need for understanding (1) how the more or less chaotic motion of microconstituents gives rise to collective, stable behaviour at the macro level, and (2) indirect effects – how behaviour in one part of the system can affect other more remote parts. For example, how do the mechanics and fluctuations of bio-molecules contribute to the organisation and functioning of a biological system, and, in economics and financial markets, how do the interactions among participants lead to complex yet stable (or unstable) markets? In both cases we have multiple components interacting in a way that generates macro properties and dynamical behaviour.

Answers to these questions ultimately involve an examination of the way that mathematics forms an integral part of the theoretical explanation. Advances in computational modelling have increased our ability to understand complex dynamical behaviour, while knowledge of certain types of complex systems, in particular those in condensed matter or statistical physics, financial markets and QFT, has been greatly aided through the use of RG methods. These involve techniques to reduce the complexity of problems for systems with large numbers of interacting constituents. Indeed, it was the introduction of RG methods (Kadanoff, 1966, 2013; Wilson, 1971, 1983) that made the development of numerical solutions in the form of Monte Carlo simulations possible for many types of complex behaviour.

What, then, is the connection between these mathematical methods and explanations that provide physical understanding? Much of the recent literature in the philosophy of mathematics (Baker, 2005, 2009, 2012; Bangu, 2008, 2008a, 2009, 2012; Batterman, 2002, 2010; French and Bueno, 2012; Leng, 2005, 2010; Pincock, 2007, 2011a, 2011b, 2012; Pincock et al., 2012; Shapiro, 1997; Steiner, 1978, 1998) has focused attention on the question of what exactly constitutes a mathematical explanation of a physical fact. The question is important not only because much of contemporary science is expressed in mathematical language but also because new phenomena are sometimes predicted on the basis of mathematics alone (e.g. the π meson particle). In that context mathematics functions as the engine of discovery, so it becomes increasingly important to determine whether, or to what extent, mathematics can embody physical information. Instead of attempting to provide necessary and sufficient conditions for what counts as a mathematical explanation – an extremely difficult task – I want to change the orientation by calling attention to specific examples of the way that mathematics in fact provides genuine physical information. Success in this context is characterised by the ability to deliver information that could not (at the time) be furnished by physical theories, hypotheses or models, and emanates solely from a mathematical method or framework. So, the question is how, over and above the calculation or prediction of numerical values for specific phenomena or effects, RG methods can facilitate this process.

RG methods became widely recognised through their employment in statistical mechanics (Kadanoff, 1966, 2013; Wilson, 1971, 1983), where they were used to investigate changes in a physical system viewed at different length/distance scales. The notion of a distance scale is important in statistical physics because theory tells us that phase transitions, the point where, for example, magnets become superconductors, occur only in infinite systems – at what is called the thermodynamic limit where the number of particles N goes to infinity, $N\rightarrow\infty$. A defining characteristic of a phase transition is an abrupt change, which, in mathematical terms, is noted by a singularity, meaning that the thermodynamic observables (represented mathematically) characterising the system are not defined or 'well-behaved' – they are infinite or not differentiable. All thermodynamic observables are partial derivatives of the partition function which describes the statistical properties of a system in thermodynamic equilibrium. This includes functions of temperature and other parameters, such as the volume enclosing a gas. Hence, a singularity in the partition function is required to obtain a singularity in the thermodynamic function. Although the partition function is analytic, it is possible for systems with infinite N to display singular behaviour for non-vanishing partition functions. Consequently, a phase transition requires a statistical mechanical system that is infinite in extent and has an infinite number of different integration variables. Physically we understand this by referring to the fact that the behaviour of a system near a phase transition (critical point) is dominated by correlated fluctuations among particles that extend over very large (∞) spatial regions. This results in the appearance of new types of macro behaviour that are largely insensitive to their initial microscopic arrangements. The computational problem, however, is how is it possible to narrow down the relevant parameters for a system that has an infinite number of constituents or variables that need to be taken account of?

Using a type of coarse graining analysis, RG methods can overcome the challenge for the physicist or mathematician by facilitating the reduction of an infinite number of particles into a tractable calculation. But, this appeal to infinite systems gives rise to a serious philosophical problem: because the phase transitions we observe (e.g. in superconductors) occur in finite systems questions arise as to how RG can provide physical information given that it requires $N\rightarrow\infty$, which seems decidedly 'non-physical'. Hence, in what sense are we really explaining the behaviour of finite systems by appealing to infinite ones? These problems are multidimensional in that they have a physical, mathematical and philosophical component. Indeed, some have argued that the appeal to infinite systems and the reduction of an infinite number of variables via RG, techniques widely used in physics, are not legitimate forms of explanation at all (Callender, 2001; Earman, 2004). I disagree with this latter point, and an ongoing part of my research is to clarify the specific sense in which the mathematics of RG plays an explanatory role in our understanding of certain types of systems.

In addition to their use in physics, RG methods have been used more recently in the treatment of certain aspects of financial markets. The analogy with statistical mechanics is based on the fact that the statistical regularities found in financial markets' time series are similar to those displayed in physical systems at critical point (Stanley et al., 1999, 1999a). In both cases there are a large number of interacting constituents (particles and agents) whose behaviour exhibits certain universal properties, properties that are independent of specific features of the individual constituents. Part of the power of RG methods is their ability to illustrate features of universal behaviour in different types of complex systems. But, as in the physics case, applications of RG in economics is not without problems. Specifically, why should we think that interesting aspects of economic behaviour can be captured by methods that ignore specific features of the agents by treating them in the same manner as molecules in a gas.

My goal, then, is to furnish a pervasive and epistemically robust role for RG methods, one that clarifies their explanatory power in enabling us to understand structural similarities that underlie the explanation and prediction of behaviour in a variety of complex systems. In order to get a clearer sense of what is at stake in RG methods and the significance of their diverse applicability, let me briefly highlight the important aspects of their use in two very different areas of physics, areas that seem to share no common features, and their success in econophysics.

Mathematics, Complexity and Explanation in Physics

If we start from a microscopic scale with the goal of explaining a system's evolution at larger scales, two problems need to be addressed. First, the scale of interest needs to be specified; second, one needs to show that the more or less chaotic motion of the elements will contribute to the collective phenomena observed at the macro scale. This, of course, is also part of the overall characterisation of emergence which is often associated with (among other things) the way that stable, complex properties arise out of a multiplicity of relatively simple interactions. In these cases specifying the relationship between the microscopic and macroscopic scales is often a difficult problem. Describing a complex system at the microscopic level requires an enormous amount of information, typically more than one is capable of handling. Consequently, the macro description necessitates the strong compression of data which effectively renders much of the microscopic information irrelevant for large scale behaviour. However, in order to deal effectively with complex systems one must still choose which relevant quantities or degrees of freedom are necessary for the problem at hand. The second problem involves establishing relations among these quantities that will generate predictions regarding their evolution. Unfortunately the motions of both the relevant and irrelevant degrees of freedom in a complex system are usually strongly coupled, which means that accurate predictions will automatically involve an accurate determination of all the irrelevant degrees of freedom as well – an essentially hopeless undertaking.

A solution to this problem was provided by RG methods which allow for a 'thinning down' of the degrees of freedom. These methods also facilitate linkages in behaviour of a system across many different scales and in cases where fluctuations on many different scales interact. The RG is often used in statistical physics to treat critical phenomena where we have many body systems that exceed the technical possibilities of the mathematical calculus of mechanics. The theory of critical phenomena deals with continuous or second order phase transitions in macroscopic systems (e.g. magnetic transitions). Such phase transitions involve collective behaviour on large scales near the critical temperature Tc where the system changes its phase. At the transition the correlation length ξ, the range over which fluctuations in one region of space are correlated with those in another, and the scale on which collective behaviour like magnetism is observed, becomes infinite. The problem is that near Tc these systems depend on two different length scales, the microscopic scale given by atoms and the dynamically generated scale give by the correlation length which characterises macro phenomena. In many classical systems one can simply decouple these different scales and describe the physics by effective macroscopic parameters without reference to the microscopic degrees of freedom. In other words, you determine the relevant scale of interest and then use a statistical averaging procedure to deal with the stochastic variables at the micro scale.

The problem however is that the statistical averaging procedure is insufficient in this case. At critical point there is a divergence of thermodynamic quantities which indicates that an infinite number of stochastic degrees of freedom are in some sense relevant to what happens at the macro level. Because of these fluctuations on all length scales one cannot simply use a statistical averaging technique; something else is required. This is unlike the typical case where physical systems have an intrinsic scale or where other relevant scales of the problem are of the same order. In these contexts phenomena occurring at different scales are almost completely suppressed, as in the case with planetary motion where it is possible to suppress, to a very good approximation, the existence of other stars and replace the size of the sun and planets by point-like objects. And, in non-relativistic quantum mechanics we can ignore the internal structure of the proton when calculating energy levels for the hydrogen atom. However, in the case of critical phenomena divergences appear when one tries to decouple different length scales. This makes it impossible to assume that a system of size L is homogenous at any length scale $l << L$. Hence, the impossibility of using statistical averaging techniques for these types of systems.

Because the statistical averaging procedures cannot accommodate the way in which inhomogeneities in the microscopic distributions contribute to large-scale cooperative behaviour, the task is to explain how short-range physical couplings can generate this type of behaviour at the macro level and how to predict it. RG methods provide a solution to such problems by determining, in a recursive manner, the effective interactions at a given scale and their relation to those at neighbouring scales.

But how, exactly, does this process work? Although it involves a technique that appears similar to averaging over small-scale correlations, it is, in fact, very different. In this case, as the length scale changes so do the values of the different parameters describing the system. Instead of calculating an average, as in statistical mechanics, the RG equations allow us to transform one set of parameters into another one with different couplings. Each transformation increases the length scale so that the transformation eventually extends to information about the parts of the system that are infinitely far away. This divergence of the correlation length means that the system 'loses memory' of its microscopic structure and begins to display new long-range macroscopic correlations. Hence, the infinite spatial extent of the system is crucial in that it determines the thermodynamic singularities included in the calculation. But again, the *way* in which this process takes place is vital for understanding the differences between RG methods and statistical averaging.

The change in the parameters is implemented by a beta function:

$$\{\tilde{J}_k\} = \beta(\{J_k\})$$

which induces what is known as an RG flow on the J-space (space governing the set of coupling constants). The phase transition is identified as the place where the RG transformations bring the flow to a fixed point, an ensemble where further iterations produce no changes in either the values of the parameters or the correlation length. These fixed points give the possible macroscopic states of the system at a large scale. So, although the correlation length diverges at critical point, using the RG equations reduces the degrees of freedom which, in effect, reduces the correlation length. What this means is that the micro physics has been 'transformed' via RG in a way that detaches it from the stable macro behaviour.

In order to understand fully how the fixed points become the focal point for explanation it is important to stress that what the fixed points do is determine the kinds of cooperative behaviour that are possible, with each type defining a universality class. The coincidence of the critical indices in very different phenomena was inexplicable prior to RG methods which were successful in showing that the differences were related to irrelevant observables – those that are 'forgotten' as the scaling process is iterated. But, the important issue is not simply the elimination of irrelevant degrees of freedom, it is the existence of cooperative behaviour characterised by the fixed points that serves as the explanatory foundation of universality.

In a general sense the elimination of unwanted degrees of freedom coincides with the suppression of information related to explanation at different levels, a strategy that is common in all areas of the physical and social sciences and is embedded in the statistical averaging procedures we find in disciplines like statistical physics and sociology. What is different in the context of RG is the way in which information is suppressed and what the end result is. A significant feature of RG is that it illustrates how, in the long wave-length/large space-scale limit, the scaling process in fact *leads* to a fixed point when the system is at a critical point, with very different microscopic structures giving rise to the same long-range behaviour. What this means is that RG equations show that critical point phenomena, despite the mathematical divergences, have an underlying order. Indeed what makes the behaviour of these phenomena predictable, even in a limited way, is the existence of certain scaling properties that exhibit universal behaviour.[1] Assuming that a fixed point is reached one can find the value that defines the critical temperature and the series expansions near the critical point provide the values of the critical indices.[2] In that sense RG methods provide us with physical information concerning how and why different systems exhibit the same behaviour near critical point. They determine these universality classes by proving the existence and universality of scaling laws, laws that provide the mathematical foundation for observed experimental behaviour.

One of the highlights of RG is that it illustrates how the same fixed point interactions are capable of describing a number of different types of systems. For instance, in condensed matter physics, systems as diverse as liquids and magnets exhibit the same type of critical-point behaviour (behaviour at phase transitions), sharing the same values for critical exponents and fixed points, a phenomenon called universality. Although universality was known experimentally, prior to the introduction of RG there was no explanation as to why or how it happened. While it is tempting to see RG methods as a technique for simply calculating the values of these critical exponents/ fixed points, my claim is that much more is involved. From the perspective of mathematical explanation what this means is that the significance of RG methods involves more than just the elimination of irrelevant degrees of freedom via the coarse graining procedure; the cooperative behaviour defined via the fixed points is crucial for understanding the explanatory power of RG in connection with complex systems.

One of the features I want to emphasise as 'physical' in explanations produced by RG is the notion of structural similarity and what this means. Meeting that challenge requires an investigation into the foundations of the RG and a comparison of its application in QFT and statistical physics (Gell Mann and Low, 1954; Weinberg, 1983). Although its use in these two very different domains suggests, prima facie, a structural similarity between them, the basis for such a similarity is not immediately obvious due to the fact that they deal with radically different types of phenomena (high and low energy). While this notion of 'same structure' is notoriously difficult to flesh out (French and Ladyman, 2003; Ladyman, 2002; Psillos, 2001, 2006; Van Fraassen, 2006, 2007) I want to claim that it involves more than simply a formal analogy between the two domains (Fraser, unpublished). Part of the argument for this point involves showing that QFT and statistical physics, regardless of their subject matter, involve similar kinds of theories, with the phenomena exhibiting similar behaviour patterns. In the case of critical phenomena one is interested only in long distance, low-energy behaviour where very short wave numbers are integrated out. In QFT the renormalization scheme is used to provide an ultraviolet cutoff at high energies. However, why should scale invariance of the sort found in QFT be important in cases of phase transitions? A very preliminary answer involves thinking of these differences in terms of their similarities: in statistical physics the grouping together of the variables referring to different degrees of freedom induces a transformation of the statistical ensemble describing the thermodynamic system. Or, one can argue in terms of a transformation of the Hamiltonian. Regardless of the notation, what we are interested in is the successive applications of the transformation that allow us to probe the system over large distances. In the field theoretic case, we do not change the 'statistical ensemble' but the stochastic variables do undergo a local transformation whereby one can probe the region of large values of the fluctuating variables. Using the RG equations, one can take this to be formally equivalent to an analysis of the system over large distances.

This formal similarity also provides some clues to why RG can be successfully applied to such diverse phenomena. In statistical physics, reducing the number of degrees of freedom with RG amounts to establishing a correspondence between one problem having a given correlation length and another whose length is smaller by a certain factor. In cases of relativistic quantum field theories choosing the appropriate renormalization scale, the logarithms that appear in perturbation theory will be minimised because all the momenta will be of the order of the chosen scale. We can think of a change in each of these scales as analogous to a phase transition where the different phases depend on the values of the parameters, with the RG allowing us to connect each of these different scales. So, regardless of whether you are integrating out very short wave numbers or using it to provide an ultraviolet cutoff, the effect is the same in that you are getting the right degrees of freedom for the problem at hand. Hence, because the *formal nature* of the problems is similar in these two domains, one can see why the RG method is so successful despite the dissimilarities between the phenomena themselves.

Econophysics and RG Explanations

The kind of long-range ordering or cooperative behaviour exhibited by statistical mechanical systems is one of the hallmarks of complex systems generally, in particular financial markets. Econophysics, the study of the dynamical behaviour of economic systems using techniques from physics, was developed in response to a number of factors (Mantegna et al., 1999; Stanley, 1999, 1999a; Stanley et al., 1999; Weatherall, 2013). Movement of prices was traditionally modelled on the concept of a random walk, using the assumption of the Gaussian character of a stochastic process. As a result the movement was considered memory-less with the negligible effects of large deviations exponentially screened in the Gaussian normal distributions. But this made it impossible to deal with wildly fluctuating markets. Also, increased computer power increased the speed and range of transactions dramatically, resulting in an amplification of the fluctuations. Finally, large amounts of financial data made possible by increased computational power meant that economies and markets began to watch each other more closely. As a result, nontrivial couplings leading to nonlinearities began to appear in economic systems. Traditional methods of analysis which emphasised homogeneous agents and equilibrium were insufficient for dealing with phenomena that depended on large numbers of heterogeneous agents and far-from-equilibrium situations; the type that give rise to emergent, complex behaviour. In other words, the mathematics at the foundation of economic theories was unable to incorporate the large-scale correlations associated with extreme or 'critical' behaviour often seen empirically in financial markets (as well as in phase transitions in statistical physics).

The danger of simplifying the underlying complexity and nonlinearity to the point where a treatable model could be constructed is that very small modifications can result in the loss of important information or introduce artificial effects. In physics, RG was developed as a response to the inadequacies of mean field theory which involved an averaging process that replaced all interactions to any one body with an average interaction, thereby reducing a multi-body problem into an effective one-body problem. This kind of averaging formed the basis for the Black-Scholes model which calculated the future value of a derivative security by taking averages in an assumed statistical market environment. The fluctuations and correlations that might exist over the entire market were ignored, with disastrous results. This is part of the phenomenon known as 'fat tails' in the (non-Gaussian) distribution where there are many events or values that stray widely from the average, giving more frequent high and low values. In financial markets this can be explained in terms of the type of universal, scaling properties (i.e. scale invariant over many orders of magnitude in the data) found in systems in statistical physics. It arises from the tendency of individual market competitors, or aggregates of them, systematically to exploit prevailing micro trends like rising or falling prices. The 'fat tails' are mathematically important, because they comprise the risks which may be negligible but which may also need to be taken account of, especially in critical situations like panic reactions to price fluctuations (Rickles, 2011). This significant deviation from the normal (Gaussian) distributions characteristic of models like the Black-Scholes formalism can only be effectively treated using RG methods. Not only do they provide a way of taking account of the long-range correlations that involve large amounts of data, they allow us to focus on only those parameters that are insensitive to changes in micro behaviour and responsible for macro regularities. In other words, the situation is exactly analogous to critical behaviour in statistical physics.

My ongoing research involves showing how the success of RG methods in these different contexts suggests a type of underlying order that is characteristic of critical behaviour in different kinds of complex systems, an ordering that is both illustrated and explained using the mathematics of RG methods. Developing a framework for mathematical explanation in these contexts allows us to appreciate not only the power of mathematics to provide physical information but also the ways in which many different types of complex systems nevertheless share a common foundation.

Notes

[1] I have not mentioned the issue of reduction in my discussion of RG, primarily because it is not immediately relevant for the general point I want to make. However, for an excellent treatment of the relation between reduction and renormalization see Batterman (2010).

[2] Phase transitions in thermodynamic systems are associated with the emergence of power-law distributions of certain quantities whose exponents are referred to as the critical exponents of the system.

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6	Donald MacKenzie	Making Things the Same: Gases, Emission Rights and the Politics of Carbon Markets	Modelling
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9	Willard Bohn	Visual Poetry in France after Apollinaire	Modelling
10	Robert A. Skipper Jr	R. A. Fisher and the Origins of Random Drift	Darwin's Legacy
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12	Atholl Anderson	Problems of the 'Traditionalist' Model of Long-Distance Polynesian Voyaging	Modelling

2009 Volume 2

No.	Author	Title	Series
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3	Ronaldo I. Borja	Landslides and Debris Flow Induced by Rainfall	Modelling
4	Roland Fletcher	Low-Density, Agrarian-Based Urbanism: A Comparitive View	Modelling
5	Paul Ormerod	21st Century Economics	Modelling
6	Peter C. Matthews	Guiding the Engineering Process: Path of Least Resistance versus Creative Fiction	Modelling
7	Bernd Goebel	Anselm's Theory of Universals Reconsidered	Modelling
8	Roger Smith	Locating History in the Human Sciences	Being Human
9	Sonia Kruks	Why Do We Humans Seek Revenge and Should We?	Being Human
10	Mark Turner	Thinking With Feeling	Being Human
11	Christa Davis Acampora	Agonistic Politics and the War on Terror	Being Human
12	Arun Saldanha	So What *Is* Race?	Being Human
13	Daniel Beunza and David Stark	Devices For Doubt: Models and Reflexivity in Merger Arbitage	Modelling
14	Robert Hariman	Democratic Stupidity	Being Human

2010 Volume 3

No.	Author	Title	Series
1	John Haslett and Peter Challenor	Palaeoclimate Histories	Modelling
2	Zoltán Kövecses	Metaphorical Creativity in Discourse	Modelling
3	Maxine Sheets-Johnstone	Strangers, Trust, and Religion: On the Vulnerability of Being Alive	Darwin's Legacy

No.	Author	Title	Series
4	Jill Gordon	On Being Human in Medicine	Being Human
5	Eduardo Mendieta	Political Bestiary: On the Uses of Violence	Being Human
6	Charles Fernyhough	What is it Like to Be a Small Child?	Being Human
7	Maren Stange	Photography and the End of Segregation	Being Human
8	Andy Baker	Water Colour: Processes Affecting Riverine Organic Carbon Concentration	Water
9	Iain Chambers	Maritime Criticism and Lessons from the Sea	Water
10	Christer Bruun	Imperial Power, Legislation, and Water Management in the Roman Empire	Water
11	Chris Brooks	Being Human, Human Rights and Modernity	Being Human
12	Ingo Gildenhard and Andrew Zissos	Metamorphosis - Angles of Approach	Being Human
13	Ezio Todini	A Model for Developing Integrated and Sustainable Energy and Water Resources Strategies	Water
14	Veronica Strang	Water, Culture and Power: Anthropological Perspectives from 'Down Under'	Water
15	Richard Arculus	Water and Volcanism	Water
16	Marilyn Strathern	A Tale of Two Letters: Reflections on Knowledge Conversions	Water
17	Paul Langley	Cause, Condition, Cure: Liquidity in the Global Financial Crisis, 2007–8	Water
18	Stefan Helmreich	Waves	Water
19	Jennifer Terry	The Work of Cultural Memory: Imagining Atlantic Passages in the Literature of the Black Diaspora	Water
20	Monica M. Grady	Does Life on Earth Imply Life on Mars?	Water
21	Ian Wright	Water Worlds	Water
22	Shlomi Dinar, Olivia Odom, Amy McNally, Brian Blankespoor and Pradeep Kurukulasuriya	Climate Change and State Grievances: The Water Resiliency of International River Treaties to Increased Water Variability	Water
23	Robin Findlay Hendry	Science and Everyday Life: Water vs H_2O	Water

2011 Volume 4

No.	Author	Title	Series
1	Stewart Clegg	The Futures of Bureaucracy?	Futures
2	Henrietta Mondry	Genetic Wars: The Future in Eurasianist Fiction of Aleksandr Prokhanov	Futures
3	Barbara Graziosi	The Iliad: Configurations of the Future	Futures
4	Jonathon Porritt	Scarcity and Sustainability in Utopia	Futures
5	Andrew Crumey	Can Novelists Predict the Future?	Futures
6	Russell Jacoby	The Future of Utopia	Futures
7	Frances Bartkowski	All That is Plastic… Patricia Piccinini's Kinship Network	Being Human
8	Mary Carruthers	The Mosque That Wasn't: A Study in Social Memory Making	Futures
9	Andrew Pickering	Ontological Politics: Realism and Agency in Science, Technology and Art	Futures
10	Kathryn Banks	Prophecy and Literature	Futures
11	Barbara Adam	Towards a Twenty-First-Century Sociological Engagement with the Future	Futures
12	Andrew Crumey and Mikhail Epstein	A Dialogue on Creative Thinking and the Future of the Humanities	Futures
13	Mikhail Epstein	On the Future of the Humanities	Futures

No.	Author	Title	Series
2012 Volume 5			
1	Elizabeth Archibald	Bathing, Beauty and Christianity in the Middle Ages	Futures II
2	Fabio Zampieri	The Holistic Approach of Evolutionary Medicine: An Epistemological Analysis	Futures II
3	Lynnette Leidy Sievert	Choosing the Gold Standard: Subjective Report vs Physiological Measure	Futures II
4	Elizabeth Edwards	Photography, Survey and the Desire for 'History'	Futures II
5	Ben Anderson	Emergency Futures	Futures
6	Pier Paolo Saviotti	Are There Discontinuities in Economic Development?	Futures II
7	Sander L. Gilman	'Stand Up Straight': Notes Toward a History of Posture	Futures II
8	Meredith Lloyd-Evans	Limitations and Liberations	Futures II
2013 Volume 6			
1	David Martin-Jones	The Cinematic Temporalities of Modernity: Deleuze, Quijano and *How Tasty was my Little Frenchman*	Time
2	Robert Levine	Time Use, Happiness and Implications for Social Policy: A Report to the United Nations	Time
3	Andy Wood	Popular Senses of Time and Place in Tudor and Stuart England	Time
4	Robert Hannah	From Here to the Hereafter: 'Genesis' and 'Apogenesis' in Ancient Philosophy and Architecture	Time
5	Alia Al-Saji	Too Late: Racialized Time and the Closure of the Past	Time
6	Simon Prosser	Is there a 'Specious Present'?	Time
2014 Volume 7			
1	Robert Fosbury	Colours from Earth	Light
2	Mary Manjikian	Thinking about Crisis, Thinking about Emergency	Time
3	Tim Edensor	The Potentialities of Light Festivals	Light
4	Angharad Closs Stephens	National and Urban Ways of Seeing	Light
5	Robert de Mello Koch	From Field Theory to Spacetime Using Permutations	Time
6	Jonathan Ben-Dov	What's In a Year? An Incomplete Study on the Notion of Completeness	Time
7	Lesley Chamberlain	Clarifying the Enlightenment	Light
8	Fokko Jan Dijksterhuis	Matters of Light. Ways of Knowing in Enlightened Optics	Light
2015 Volume 8			
1	Valerie M. Jones	Mobile Health Systems and Emergence	Emergence
2	Stéphanie Portet	Studying the Cytoskeleton: Case of Intermediate Filaments	Modelling
3	Peter Cane	Two Conceptions of Constitutional Rights	Emergence
4	Nathan J. Citino	Cultural Encounter as 'Emergence': Rethinking US-Arab Relations	Emergence
5	N. Katherine Hayles	Nonconscious Cognition and Jess Stoner's *I Have Blinded Myself Writing This*	Emergence
6	Alice Hills	Waiting for Tipping Points	Emergence

Insights

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