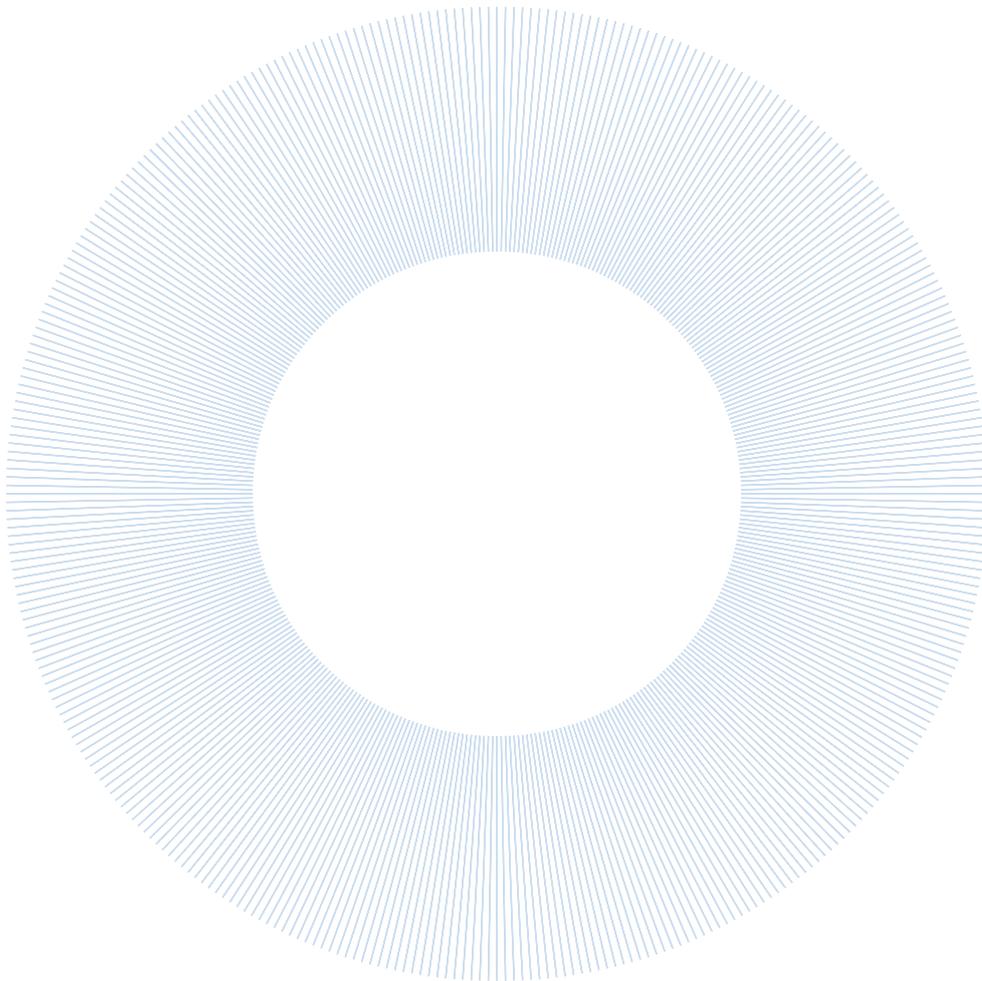


From Field Theory to Spacetime Using Permutations



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Volume 7

2014

Number 5

ISSN 1756-2074

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FROM FIELD THEORY TO SPACETIME USING PERMUTATIONS

The AdS/CFT claims an exact equivalence between certain theories of quantum gravity, defined in $d+1$ dimensions, and conformal field theories defined in d dimensions. Since quantum gravity is a fundamental theory of spacetime, the AdS/CFT correspondence is expected to provide deep insights into the most basic structure of space and time. At present the correspondence is a conjecture which has passed an impressive number of tests but remains somewhat mysterious. This article suggests a set of variables with which new features of the correspondence can be explored.



Modern physics rests on two central pillars: general relativity and quantum field theory. General relativity, Einstein's non-linear theory of gravity, describes spacetime as a dynamical manifold that curves in response to the energy and momentum that is present in the spacetime (Wald, 2010). Quantum field theory describes the remaining (non-gravitational) interactions (Peskin and Schroeder, 1995). Both general relativity and quantum field theory enjoy impressive experimental confirmation. However, as has become clear from certain problems in black hole physics and from thinking about the Big Bang, a complete description of nature requires the unification of general relativity and quantum field theory into a single theoretical framework, quantum gravity. Fundamental theories of spacetime, and hence of time itself, are expected to arise from this unification. Indeed, the problem of unifying general relativity and quantum field theory is the problem of understanding the quantum nature of spacetime. The problem is remarkably difficult and a consistent complete theory of quantum gravity relevant for the description of our world is an ongoing project (Klebanov and Maldacena, 2009). In this article a speculative approach to this problem is outlined.

A promising candidate for a theory of quantum gravity is provided by string theory (Polchinski, 1998). The basic idea in string theory is to replace the elementary point particles of nature by fundamental strings which are one-dimensional objects. The different oscillation modes of these fundamental strings are supposed to give the different elementary particles that are observed in nature, one particle from each oscillation mode of the string. The strings can split and join, leading to string interactions which are supposed to match the four fundamental forces discovered in nature. String theory is remarkably constrained. Indeed, consistency of the theory determines how the fundamental strings interact. The picture that emerges is strikingly similar to general relativity and the quantum field theory description of the fundamental forces that have already proved to be useful descriptions of nature.

Within the context of string theory, the dynamics of spacetime emerges as a necessary condition for the consistency of the theory. More specifically, one is considering first quantized string theory in which the dynamics of a string is described in terms of a 1+1 dimensional quantum field theory living on the string worldsheet. Conformal invariance, one of the gauge symmetries of the worldsheet theory, requires that all of the beta functions vanish. From the worldsheet point of view, the metric of spacetime is just another coupling constant, one that is field dependent. The vanishing of the beta function associated with the metric reproduces the Einstein field equations of general relativity and fixes the spacetime dimension to be 26 for the bosonic string theory, or 10 for the superstring theory (see for example: Callan, Martinec, Perry et al., 1985).

In a sense, spacetime dynamics is not one of the fundamental ingredients put in by hand, but rather it emerges as a consistency condition of the theory.

String theory is a rather speculative, incomplete approach to quantum gravity and one might question insights derived solely from string theory. There is, however, independent evidence confirming and extending the above picture suggested by string theory. Indeed as we will review shortly, based on very general properties of gravity, 't Hooft (1993) and Susskind (1995) have proposed the holographic principle and Jacobson (1995) has developed a remarkable thermodynamical derivation of Einstein's field equations. Both of these ideas point to gravity as an emergent phenomenon.

Since black holes are exact solutions of Einstein's equations, specified completely by their mass, electric charge and angular momentum, it is somewhat natural to assume that they do not have any entropy. Jacob Bekenstein (1972) argued that this leads to a violation of the second law of thermodynamics. To see the violation, imagine that a hot gas with entropy is thrown into a black hole. Once the gas crosses the event horizon, the entropy associated with it will disappear. This contradicts the second law of thermodynamics which claims that the entropy of any system increases or stays constant. The only way to salvage the second law, is to assign to black holes an enormous entropy whose increase is greater than the entropy carried by the gas. In this way, Bekenstein argued that there is an upper bound on the entropy in a region of space. The bound is proportional to the area of the region. In particular, the black hole entropy is directly proportional to the area of the event horizon. Thus, the logarithm of the number of states of a black hole is proportional to the area of the horizon, not the volume in the interior. Taken to its logical conclusion ('t Hooft, 1993), this observation implies that the quantum gravity dynamics of any region \mathbf{R} can be described by a non-gravitational quantum field theory living on the boundary of the region.

Jacobson's argument (1995) uses three basic inputs. A non-inertial observer moving with an acceleration \mathbf{a} claims that the vacuum of quantum field theory in Minkowski spacetime is a thermal state at the Unruh temperature $\mathbf{T}=\hbar\mathbf{a}/(2\pi)$ (Unruh, 1976). Second, motivated by black hole physics (Bekenstein, 1972), it is natural to assign an entropy of $\mathbf{S}=\mathbf{A}2\pi/4\hbar\mathbf{G}$ to any causal horizon of area \mathbf{A} in a locally Minkowski patch of spacetime. By interpreting the energy exchange $\delta\mathbf{E}$ as the energy of matter flowing across the local Rindler horizon of the accelerated observer, Jacobson requires the local entropy balance relation $\delta\mathbf{S}=\delta\mathbf{E}/\mathbf{T}$. By matching the variation of the area with the focusing effect of spacetime curvature, Jacobson showed that if the three equations above hold for any local frame, then the Einstein's equations follow. This result suggests that Einstein's equations have a statistical origin and should be interpreted as equations of state for unknown underlying degrees of freedom, with the metric being a macroscopic 'coarse grained' variable. String theory provides a possible candidate for these more fundamental 'atoms of spacetime' and in the process provides a very concrete example of the holographic principle.

One of the most promising ideas to emerge from string theory in the last 15 years is the AdS/CFT correspondence (Maldacena, 1998). The correspondence claims an exact equivalence between certain theories of quantum gravity, defined in $\mathbf{d}+1$ dimensions, and conformal field theories defined in \mathbf{d} dimensions. The quantum gravity theory is defined on a spacetime whose geometry is asymptotically Anti-de Sitter (AdS) spacetime – a spacetime with a constant negative curvature. Conformal field theory (CFT) is a quantum field theory that enjoys conformal symmetry. The higher dimensional gravitational theory provides a holographic description of the lower dimensional quantum field theory. As discussed above, there are clear arguments that suggest that spacetime is not a fundamental concept, but that it is an emergent concept useful

as a description in certain limits of nature. The AdS/CFT correspondence gives a very concrete set-up in which to explore this idea and it is the setting for the work described in this article.

The example of the AdS/CFT correspondence that we will make use of relates $N_s=4$ super Yang-Mills theory to IIB string theory on asymptotically $AdS_5 \times S^5$ spacetime. The Yang-Mills theory has gauge group $U(N)$ and coupling constant g_{YM}^2 . We are most interested in the dynamics of the field theory in the 't Hooft limit, obtained by taking N to ∞ , $g_{YM}^2 \propto \lambda$ holding the so-called 't Hooft coupling $\lambda = g_{YM}^2 N$ fixed. The coupling of the string theory g_s is related to the coupling of the field theory as $g_s = g_{YM}^{-2}$. The radius of curvature of the AdS space R , in units of the string length, is $R^4 = g_{YM}^{-2} N$. The limit of strong coupling in the field theory (λ large) is also the regime in which the geometry of the string theory is nearly flat. Thus, the limit in which we can neglect curvature corrections leading to relatively simple stringy dynamics maps onto the very difficult problem of strongly coupled quantum field theory. Conversely, weakly coupled field theory (small λ) corresponds to highly curved string geometries. Thus, the limit of simple field theory dynamics maps into strongly coupled string interactions. This feature of the AdS/CFT correspondence, that difficult dynamical problems are mapped into rather simple dual dynamics, makes the correspondence at once extremely useful and difficult to prove.

The correspondence comes equipped with a dictionary that tells you how to relate the physics of the quantum field theory to the physics of quantum gravity. Some of the entries of the dictionary are determined by relating the global symmetries enjoyed by the two theories. Although the symmetry groups must match, the explicit realization of the group can differ. As an example of what the dictionary contains, operators in the conformal field theory are dual to states in the quantum gravity. Further, operator dimensions are equal to the energies of the dual states. It is useful to organize the correspondence between gravity states and conformal field theory operators according to the energy of the states which is the dimension of the operators. Operators with a dimension of order 1 are dual to point like gravitons (Witten, 1998). Operators with a dimension of order \sqrt{N} are dual to strings (Berenstein, Maldacena and Nastase, 2002). Operators with a dimension of order N are dual to giant gravitons (McGreevy, Susskind and Toumbas, 2000). Operators with a dimension of order N^2 are dual to new geometries (Lin, Lunin and Maldacena, 2004). The organization according to dimension is thus an organization according to different semi-classical states we can describe in the string theory.

In addition, the field theory methods that can be used are also naturally organized. The large N limit of operators with a dimension that grows no faster than \sqrt{N} is captured by summing the planar diagrams. In this limit, expectation values of gauge invariant operators factorize, which is a signal that the path integral is being dominated by a single configuration. Indeed, let a general set of observables be represented by the operators O_A . The system can be in a number of distinct but definite states, indexed by the label i . The operator O_A takes the value $O_A(i)$ in state i . The probability that the system is in state i is $\mu(i)$. Of course, $\sum_i \mu(i) = 1$ and $\mu(i) \geq 0$. The expectation value of an operator is given by the usual expression

$$\langle O_A \rangle = \sum_i \mu(i) O_A(i)$$

Factorization is the statement that

$$\langle O_A O_B \dots O_Z \rangle = \langle O_A \rangle \langle O_B \rangle \dots \langle O_Z \rangle$$

$$\sum_i \mu(i) O_A(i) O_B(i) \dots O_Z(i) = \sum_i \mu(i) O_A(i) \sum_j \mu(j) O_B(j) \dots \sum_k \mu(k) O_Z(k)$$

These relations hold for an arbitrary product of observables. The natural interpretation of these relations is that $\mu(\mathbf{i})=0$ except for a single configuration \mathbf{i}^* which has $\mu(\mathbf{i}^*)=1$. Thus, only a single configuration enters and we are in a classical limit of the theory. Thanks to the AdS/CFT correspondence we know the classical system whose dynamics reproduces the large \mathbf{N} gauge theory – it is type IIB string theory on the $\text{AdS}_5 \times S^5$ background. Working directly on the gauge theory side, in problems that amount to the dynamics of a single matrix, it is possible to reduce the large \mathbf{N} dynamics to eigenvalue dynamics which can be solved exactly. The eigenvalues provide an efficient parameterization of the dynamics: there are only \mathbf{N} eigenvalues. Since the fluctuations are of size $1/\mathbf{N}^2$, fluctuations in the eigenvalues can at most produce an effect that is of order $\mathbf{N} \times (1/\mathbf{N}^2)=1/\mathbf{N}$ which vanishes in the large \mathbf{N} limit. The eigenvalue dynamics thus become classical dynamics. It would seem that reducing the dynamics of the super Yang-Mills theory would allow a rather direct comparison to the dual gravitational dynamics. This logic is beautifully illustrated in the 1/2-BPS sector (Lin, Lunin and Maldacena, 2004). It is however a formidable task to extend this analysis to more than one matrix.

The usual argument that only the planar diagrams contribute at large \mathbf{N} is based on the observation that non-planar diagrams are suppressed by a factor \mathbf{N}^{-2k} with $k \geq 1$. The number of Feynman diagrams contributing for operators that have a dimension that grows as \mathbf{N} or as \mathbf{N}^2 is itself \mathbf{N} dependent. This \mathbf{N} dependence tends to enhance the non-planar diagrams, simply because there are so many more non-planar diagrams than there are planar diagrams. For operators with a dimension that grows as \mathbf{N} or \mathbf{N}^2 , the huge number of non-planar diagrams overpowers the \mathbf{N}^{-2k} suppression and non-planar diagrams simply cannot be neglected (Balasubramanian, Berkooz, Naqvi et al., 2002). For this reason, we will refer to these limits as large \mathbf{N} but non-planar limits of the theory. New techniques to study this limit of the theory are needed.

At the free field limit, the Feynman diagrams that arise come from a simple application of Wick's theorem. Wick's theorem tells us to pair fields and replace each pairing by its Wick contraction – a well-defined function depending on the field's location in spacetime, spin and gauge group indices. The sum over all pairings is a combinatoric problem that can be organized very efficiently using the group representation theory of both the symmetric group and the unitary group, as well as the relation between the two which is known as Schur-Weyl duality. The result of this exercise is a basis for the local operators of the theory. The particular operators that we will use are known as restricted Schur polynomials (Bhattacharyya, Collins and de Mello Koch, 2008). These operators diagonalize the two-point function and display weak mixing when quantum corrections are included. Further, diagonalizing the two-point function in the presence of interactions has been reduced to the problem of decoupled harmonic oscillators which can be solved explicitly (de Mello Koch, Desein, Giataganas et al., 2011). These oscillators are labelled by graphs with a fixed number of nodes and with directed line segments stretching between nodes of the graph (de Mello Koch and Ramgoolam, 2012). The operators with a dimension that grows as \mathbf{N} that have been studied, are dual to giant gravitons, certain objects in the string theory (McGreevy, Susskind and Toumbas, 2000). Giant gravitons are three (spatial) dimensional objects. The worldvolume dynamics of giant gravitons at low energy is itself a Yang-Mills theory. Excitations of the giant gravitons are described by open strings that end on the giants. As a consequence of the Gauss Law, the allowed open string configurations are tightly constrained (Balasubramanian, Berenstein, Feng et al., 2005). Rather remarkably, the graphs introduced above that emerge to label the decoupled oscillators are nothing but the allowed open string configurations of a system of giant gravitons. Clearly, these methods are powerful and allow direct study of the large \mathbf{N} but non-planar limits of the $\mathbf{N}_s=4$ super Yang-Mills theory.

The utility of the new group representation theory methods has been established. They have played the role of a powerful technical tool. A more ambitious approach would be to try to

use the new methods as a starting point by which to explore the general structure of the AdS/CFT correspondence. As mentioned above, the eigenvalues of a single matrix provide a set of variables that become classical in the large \mathbf{N} limit. It is natural to try to connect these variables to the dual gravitational theory and this idea has been successfully implemented (Lin, Lunin and Maldacena, 2004). The eigenvalue variables become classical because the number of eigenvalues (\mathbf{N}) times the fluctuation in each eigenvalue ($1/\mathbf{N}^2$) goes to zero as \mathbf{N} to ∞ . Is there a similar argument that can be used either to establish or deny the utility of the restricted Schur polynomials? Our approach in constructing this argument will be to consider a specific large \mathbf{N} but non-planar limit relevant to a definite semi-classical configuration.

First, it is useful to describe how restricted Schur polynomials are labelled in general. Irreducible representations of the symmetric group play a central role in the construction of the restricted Schur polynomials. The irreducible representations of the symmetric group are labelled by Young diagrams. A Young diagram is a collection of boxes collected into rows and then stacked into a triangular shape, with the longest row at the top of the diagram. These Young diagrams have at most \mathbf{N} rows, so that a Young diagram is completely specified by giving the \mathbf{N} numbers which record the number of boxes in each row. The restricted Schur polynomials are labelled by a collection of Young diagrams (one for each species of field out of which our operator is composed) as well as a set of multiplicity labels. Denote the number of species of field out of which our operator is composed by \mathbf{n}_s . Each Young diagram has \mathbf{N} rows, so that ignoring multiplicity labels for now, the restricted Schur polynomial is specified by giving $\mathbf{n}_s \mathbf{N}$ numbers. If we restrict to $\mathbf{n}_s=1$, there are no multiplicity labels and we have to specify a total of \mathbf{N} row lengths to specify the operator. This matches the number of eigenvalues, and in fact, there is a connection between row lengths and eigenvalues (de Mello Koch, 2008). A related comment is that the eigenvalue dynamics is the dynamics of non-interacting fermions and the row lengths can be translated into a very specific free fermion wave function. The fluctuations in row lengths are closely related to the fluctuations in the eigenvalues (Okounkov, 2000), so that we again have \mathbf{N} variables with fluctuation of size $1/\mathbf{N}^2$. This last observation in particular makes it clear that in general, if we have any hope that we can recover classical dynamics in the new restricted Schur variables, we must restrict ourselves to operators for which \mathbf{n}_s is fixed as \mathbf{N} to ∞ . To go further we need a careful description of the multiplicity labels. We will do this by focusing on specific semi-classical limits.

Consider first the large \mathbf{N} limit of the sector of the theory comprising operators with dimension of order \mathbf{N} that are dual to a system of p giant gravitons, with p held fixed as we take \mathbf{N} to ∞ . The labels for these operators are Young diagrams with a total of order \mathbf{N} boxes and exactly p rows or columns. Thus, generically, each long row or column of the Young diagram contains order \mathbf{N} boxes. To simplify the analysis even further, we will assume that the restricted Schur polynomials are built from two species of fields. We will further consider operators built with \mathbf{n} \mathbf{Z} fields (\mathbf{n} scales as \mathbf{N}) and \mathbf{m} \mathbf{Y} fields (\mathbf{m} is held fixed as we take \mathbf{N} to ∞). The advantage of considering this setting is that the multiplicity labels are known to be states in a $\mathbf{U}(p)$ representation labelled with a Young diagram that contains m boxes. This implies that the multiplicity labels run over no more than p^m values. Consequently, the total number of labels for the restricted Schur polynomial grows as \mathbf{N} when we take \mathbf{N} to ∞ . Assuming that these labels fluctuate in the same way that the eigenvalues do (this has not been proved but it seems plausible given what we know from the one matrix sector), our counting would prove that the restricted Schur polynomials describing this sector are governed by classical dynamics.

As a second example, consider operators with a dimension that grows as \mathbf{N}^2 as we scale \mathbf{N} to ∞ . These operators are dual to new spacetime geometries and consequently, this sector of the theory is one of the most interesting if we are hoping to learn something fundamental about

spacetime. The Young diagrams labelling these restricted Schur polynomials generically have \mathbf{N} rows. We will again build our restricted Schur polynomials from two species of fields: \mathbf{n} \mathbf{Z} fields (\mathbf{n} scales as \mathbf{N}^2) and \mathbf{m} \mathbf{Y} fields (\mathbf{m} is held fixed as we take \mathbf{N} to ∞). We have a very dilute gas of \mathbf{Y} s so that we can label the operator by (i) the row lengths of the Young diagram, and (ii) the rows in which the impurities \mathbf{Y} appear. These are again $\mathbf{O}(\mathbf{N})$ labels, so that if we again assume that these labels fluctuate in the same way that the eigenvalues themselves fluctuate, we would again have classical dynamics for the restricted Schur polynomials.

In the proposal outlined above, the row lengths are to have a very direct interpretation as variables of the dual gravitational theory. One would expect then that some locality on the Young diagram labels emerges. Of course, this locality is strictly only expected at large 't Hooft coupling λ which is difficult to test. At weak coupling we can look for and do see signals of this locality (de Mello Koch, 2008). It would be very interesting to explore this issue at strong coupling which is the limit relevant for smooth geometries in the dual gravity.

Assuming that the identification of the dynamical variables proposed above is a sensible idea, one can ask how this is to be accomplished. More precisely, how do you rewrite the dynamics of the theory in terms of the row lengths and multiplicity indices of the Young diagram labels of the restricted Schur polynomial operators? As a first step towards answering this question, it is useful to ask what the physical interpretation of the row length labels and the multiplicity indices are. This interpretation depends on the specific class of operators we choose to study. Consider first the class of operators that have a dimension that grows as \mathbf{N} when we take the \mathbf{N} to ∞ limit. The operators that have a clear physical interpretation in the dual gravitational theory have a fixed order $\mathbf{1}$, say \mathbf{p} , number of rows. These operators are dual to a system of \mathbf{p} giant gravitons. We will imagine that the number of \mathbf{Z} s grows as \mathbf{N} when we take \mathbf{N} to ∞ and that there are a small number (order $\mathbf{1}$) of \mathbf{Y} fields. In this case, the row lengths of the Young diagram labels correspond to radii (or equivalently angular momenta) of individual giant gravitons in the system. In the large \mathbf{N} limit these become continuous parameters playing the role of a collective coordinate for the giant graviton. The multiplicity labels remain a discrete label, even at large \mathbf{N} . This discrete label selects a specific configuration from the many that would be consistent with the Gauss law. In settings where we allow more species of matrices and more (in general order \mathbf{N}) fields of each species, we expect further continuous parameters to emerge. In the general set up, these labels should give a complete set of collective coordinates for the giant graviton. The \mathbf{Y} fields become the open string excitations of the giant graviton setting. The description we have employed up to now has treated each \mathbf{Y} field as a 'short' open string that has no internal structure. This can be and should be extended, by replacing each single \mathbf{Y} field by an open string word. The dynamics of this open string word would then be mapped to an open spin chain (Minahan and Zarembo, 2003) and this word would itself have labels that become continuous at large \mathbf{N} .

Consider now the class of operators whose dimension grows as order \mathbf{N}^2 when we take \mathbf{N} to ∞ . The operators that have the clearest physical interpretation belong to the so-called 1/2-BPS sector of the theory. In this case, the restricted Schur polynomials reduce to the much simpler Schur polynomials which are labelled by a single Young diagram. Using nothing more than symmetry arguments (Lin, Lunin and Maldacena, 2004) have shown that the supergravity equations determining the geometry amount to solving the (linear) Laplace equation for a function $\mathbf{z}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{y})$ in the half space $\mathbf{y} > \mathbf{0}$ of the three-dimensional space with coordinates $(\mathbf{y}, \mathbf{x}_1, \mathbf{x}_2)$. The boundary condition for this Laplace equation is given by colouring the $\mathbf{y} = \mathbf{0}$ plane black and white. On white regions we set $\mathbf{z}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{0}) = \mathbf{1}/\mathbf{2}$. On black regions we set $\mathbf{z}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{0}) = -\mathbf{1}/\mathbf{2}$. The set of all boundary conditions is the complete space of supergravity solutions (moduli space). Quantizing this moduli space leads to a sensible dynamical picture, so that these boundary

conditions can be considered as a set of dynamical variables for the gravity problem. The Schur polynomial maps to a very specific boundary condition, as we now explain. Every Schur polynomial corresponds to colouring the $y=0$ plane using a set of concentric annuli. Moving radially out from the centre of this collection of annuli corresponds to traversing the right edge of the Young diagram. The N th row of the Young diagram maps to the centres of the annuli. This again illustrates a rather direct and explicit equivalence between the labels of our gauge theory operator (again the row lengths of a Young diagram) and the dynamical variables of the gravity description. Of course, the set-up we are describing corresponds to single matrix dynamics and the problem may well be too simple to illustrate general lessons. To clarify this issue further and provide non-trivial support for the proposal we are putting forward, it is important to extend our $\frac{1}{2}$ BPS analysis to the $\frac{1}{4}$ BPS sector. In the field theory, this corresponds to considering the dynamics of two matrices which requires three Young diagrams as well as multiplicity labels. The goal then would be to elucidate how the labels of the restricted Schur polynomials are identified with dynamical variables of the dual gravitational theory. Solving this problem will teach us about 'effective eigenvalue dynamics' for two matrices and is a strong test of the idea that the dynamics will again become classical if we describe things using the row-length variables.



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1	David Martin-Jones	The Cinematic Temporalities of Modernity: Deleuze, Quijano and <i>How Tasty was my Little Frenchman</i>	Time
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3	Andy Wood	Popular Senses of Time and Place in Tudor and Stuart England	Time
4	Robert Hannah	From Here to the Hereafter: 'Genesis' and 'Apogenesis' in Ancient Philosophy and Architecture	Time
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1	Robert Fosbury	Colours from Earth	Light
2	Mary Manjikian	Thinking about Crisis, Thinking about Emergency	Time
3	Tim Edensor	The Potentialities of Light Festivals	Light
4	Angharad Closs Stephens	National and Urban Ways of Seeing	Light

Insights

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